

Quantum Hall Dynamics on von Neumann Lattice

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Quantum Hall Dynamics is formulated on von Neumann lattice representation where electrons in Landau levels are defined on lattice sites and are treated systematically like lattice fermions. We give a proof of the integer Hall effect, namely the Hall conductance is the winding number of the propagator in the momentum space and is quantized exactly as integer multiple of $\frac{e^2}{h}$ in quantum Hall regime of the system of interactions and disorders. This shows that a determination of the fine structure constant from integer quantum Hall effect is in fact possible. We present also a unified mean field theory of the fractional Hall effect for the in-compressible quantum liquid states based on flux condensation and point out that the known Hofstadter butterfly spectrum of the tight binding model has a deep connection with the fractional Hall effect of the continuum electrons. Thus two of the most intriguing and important physical phenomena of recent years, the integer Hall effect and fractional Hall effect are studied and are solved partly by von Neumann lattice representation.

I. VON NEUMANN LATTICE REPRESENTATION

A. Quantum Hall system

Quantum Hall system is a system of two dimensional electrons in strong perpendicular magnetic field, and is realized in semiconductors. The system shows intriguing physical phenomena such as the integer Hall effect and the fractional Hall effect. The Hall conductance agrees with $N \times e^2/h$ or $p/q \times e^2/h$ in finite parameter regions, hence, quantum Hall effect is used as a standard of resistance and precise determination of the fine structure constant provided the above relations are exact. It is a theoretical issue to find out if the above relations are exact or not.

The fractional Hall effect shows that the ground state of many electron systems of certain fractional filling is unique and is a quantum liquid of having large energy gap. The energy gap vanishes and the ground state has an enormous degeneracy in the absence of interactions. To find the mechanism of forming this kind of incompressible liquid by interactions is another issue for the theorists.

Von Neumann lattice is a spatial lattice which is defined from a complete set of coherent states, i.e., eigenstates of annihilation operator. Von Neumann lattice representation preserves a spatial symmetry in lattice form and is useful in studying the above problems. We have used it for studying the quantum Hall systems and for solving the quantum Hall dynamics. The proof of the integer Hall effect and others have been given. [1–4,6]

In quantum Hall system, it is convenient to decompose the electron coordinates (x, y) into two sets of variables, guiding center variables and relative coordinates variables. Guiding center variables (X, Y) stand for the center coordinates of cyclotron motion and their commutation relation becomes an imaginary number that is inversely proportional to the external magnetic field. A minimum complete set of coherent states in this space has discrete complex eigenvalues and is known as von Neumann lattice representation. Relative coordinates (ξ, η) satisfy the equivalent commutation relation and the one body free Hamiltonian is proportional to the summation of squares of relative coordinates. Hence the electron has a discrete eigenvalue of energy. Each energy level is known as Landau level and its degeneracy is specified by the center variables. From commutation relation, the degeneracy per area is proportional to the magnetic field. The coherent states are defined by

$$(X + iY)|\alpha_{mn}\rangle = z_{mn}|\alpha_{mn}\rangle, \quad (1)$$

$$z_{mn} = (m\omega_x + n\omega_y)a,$$

where m, n are integers and ω_x, ω_y are complex numbers which satisfy $\text{Im}[\omega_x^* \omega_y] = 1$ and z_{mn} is a point on the lattice site in the complex plane; an area of the unit cell is a^2 . We call this lattice the magnetic von Neumann lattice. With a spacing $a = \sqrt{\frac{2\pi\hbar}{eB}}$, the completeness of the set $\{|\alpha_{mn}\rangle\}$ is ensured. [7] Fourier transformed states denoted by

$$|\alpha_{\mathbf{p}}\rangle = \sum_{m,n} e^{ip_x m + ip_y n} |\alpha_{m,n}\rangle, \quad (2)$$

are orthogonal, that is,

$$\langle \alpha_{\mathbf{p}} | \alpha_{\mathbf{p}'} \rangle = \alpha(\mathbf{p}) \sum_N (2\pi)^2 \delta(\mathbf{p} - \mathbf{p}' - 2\pi \mathbf{N}). \quad (3)$$

Here, $\mathbf{N} = (N_x, N_y)$ is a vector with integer values and $\mathbf{p} = (p_x, p_y)$ is a momentum in the Brillouin zone (BZ), that is, $|p_x|, |p_y| \leq \pi$. The function $\alpha(\mathbf{p})$ is calculated by using the Poisson resummation formula as follows:

$$\alpha(\mathbf{p}) = \beta(\mathbf{p})^* \beta(\mathbf{p}), \quad (4)$$

$$\beta(\mathbf{p}) = (2\text{Im}\tau)^{\frac{1}{4}} e^{i\frac{\tau}{4\pi} p_y^2} \vartheta_1\left(\frac{p_x + \tau p_y}{2\pi} | \tau\right), \quad (5)$$

where $\vartheta_1(z|\tau)$ is a theta function and the moduli of the von Neumann lattice is defined by $\tau = -\omega_x/\omega_y$. The magnetic von Neumann lattice is parameterized by τ . To indicate the dependence on τ , we sometimes use a notation such as $\beta(\mathbf{p}|\tau)$. For $\tau = i$, the von Neumann lattice becomes a square lattice. For $\tau = e^{i2\pi/3}$, it becomes a triangular lattice. Some properties of the above functions are presented in References . Whereas $\alpha(\mathbf{p})$ satisfies the periodic boundary condition, $\beta(\mathbf{p})$ obeys a nontrivial boundary condition

$$\beta(\mathbf{p} + 2\pi \mathbf{N}) = e^{i\phi(p, N)} \beta(\mathbf{p}), \quad (6)$$

where $\phi(p, N) = \pi(N_x + N_y) - N_y p_x$. We can define the orthogonal state which is normalized with δ -function as follows:

$$|\beta_{\mathbf{p}}\rangle = \frac{|\alpha_{\mathbf{p}}\rangle}{\beta(\mathbf{p})}. \quad (7)$$

It should be noted that the state $|\alpha_0\rangle$ is a null state, that is, $\sum_{m,n} |\alpha_{mn}\rangle = 0$, because $\beta(0) = 0$.

The Hibert space of one-particle states is also spanned by the state $|f_l \otimes \beta_{\mathbf{p}}\rangle$. We call the state $|f_l \otimes \beta_{\mathbf{p}}\rangle$ the momentum state of the von Neumann lattice. The wave function of the state $|f_l \otimes \beta_{\mathbf{p}}\rangle$ in the spatial coordinate space is given in references.

The probability density $|\langle \mathbf{x} | f_l \otimes \beta_{\mathbf{p}} \rangle|^2$ is invariant under the translation $(\tilde{x}, \tilde{y}) \rightarrow (\tilde{x} + aN_x, \tilde{y} + aN_y)$. Thus, the momentum state is an extended state.

B. Field Theoretical Formalism and Topological Formula of Hall Conductance

In the preceding section we obtain one-particle states based on the von Neumann lattice, that is, the coherent state $|f_l \otimes \alpha_{mn}\rangle$ and the momentum state $|f_l \otimes \beta_{\mathbf{p}}\rangle$. Now we develop the field theoretical formalism based on the momentum state . From now on, we denote $|f_l \otimes \beta_{\mathbf{p}}\rangle$ as $|l, \mathbf{p}\rangle$. We expand the electron field operator in the form

$$\psi(\mathbf{x}) = \int_{\text{BZ}} \frac{d^2 p}{(2\pi)^2} \sum_{l=0}^{\infty} b_l(\mathbf{p}) \langle \mathbf{x} | l, \mathbf{p} \rangle. \quad (8)$$

$b_l(\mathbf{p})$ satisfies the anti-commutation relation

$$\{b_l(\mathbf{p}), b_{l'}^\dagger(\mathbf{p}')\} = \delta_{l,l'} \sum_N (2\pi)^2 \delta(\mathbf{p} - \mathbf{p}' - 2\pi \mathbf{N}) e^{i\phi(p', N)}, \quad (9)$$

and the same boundary condition as $\beta(\mathbf{p})$. b_l^\dagger and b_l are creation and annihilation operators which operate on the many-body states. The free Hamiltonian is given by

$$\mathcal{H}_0 = \int d^2 x \psi^\dagger(\mathbf{x}) \hat{H}_0 \psi(\mathbf{x}) = \sum_l \int_{\text{BZ}} \frac{d^2 p}{(2\pi)^2} E_l b_l^\dagger(\mathbf{p}) b_l(\mathbf{p}). \quad (10)$$

The density and current operators in the momentum space $j_\mu = (\rho, \mathbf{j})$ become

$$\begin{aligned} j_\mu(\mathbf{k}) &= \int_{\text{BZ}} \frac{d^2 p}{(2\pi)^2} \sum_{l,l'} b_l^\dagger(\mathbf{p}) b_{l'}(\mathbf{p} - a\hat{\mathbf{k}}) \\ &\quad \langle f_l | \frac{1}{2} \{v_\mu, e^{ik \cdot \xi}\} | f_{l'} \rangle e^{-\frac{i}{4\pi} a\hat{k}_x (2p - a\hat{k})_y}, \end{aligned} \quad (11)$$

Here, $v^\mu = (1, -\omega_c \eta, \omega_c \xi)$, and $\hat{k}_i = W_{ij} k_j$. The explicit form of $\langle f_l | e^{ik \cdot \xi} | f_{l'} \rangle$ and W_{ij} are given in references.

The free Hamiltonian \mathcal{H}_0 is diagonal in the above basis. However, the density operator is not diagonal with respect to the Landau level index. This basis, which we call the energy basis, is convenient to describe the energy spectrum of the system. In another basis, \mathcal{H}_0 is not diagonal and the density operator is diagonal. This basis, which we call the current basis, is convenient to describe the Ward-Takahashi identity and the topological formula of Hall conductance. There is no basis in which both the Hamiltonian and the density are diagonal. This is one of peculiar features in a magnetic field.

The current basis is constructed as follows. Using a unitary operator, we can diagonalize the density operator in the Landau level indices. We define the unitary operator

$$U_{ll'}^\dagger(\mathbf{p}) = \langle f_l | e^{i\tilde{p} \cdot \xi / a - \frac{i}{4\pi} p_x p_y} | f_{l'} \rangle. \quad (12)$$

By introducing a unitary transformed operator $\tilde{b}_l(\mathbf{p}) = \sum_{l'} U_{ll'}(\mathbf{p}) b_{l'}(\mathbf{p})$, the density operator is written in the diagonal form and the current operator becomes a simple form:

$$\rho(\mathbf{k}) = \int_{\text{BZ}} \frac{d^2 p}{(2\pi)^2} \sum_l \tilde{b}_l^\dagger(\mathbf{p}) \tilde{b}_l(\mathbf{p} - a\hat{\mathbf{k}}). \quad (13)$$

\tilde{b}_l and \tilde{b}_l^\dagger satisfy the anti-commutation relation and boundary condition.

Here we show the Ward-Takahashi identity and the topological formula of Hall conductance using the current basis. The one-particle irreducible vertex part $\tilde{\Gamma}^\mu$ is connected with the full propagator by the Ward-Takahashi identity. The identity has crucial roles in the following derivation of the topological formula of Hall conductance. The Ward-Takahashi identity in this case becomes

$$\tilde{\Gamma}_\mu(p, p) = \frac{\partial \tilde{S}^{-1}(p)}{\partial p^\mu}. \quad (14)$$

In a theory without a magnetic field, Ward-Takahashi identity gives a relation that the state of the dispersion $\epsilon(p)$ moves with the velocity $\frac{\partial \epsilon(p)}{\partial p_i}$. However in a magnetic field, we can not diagonalize both the current and the energy simultaneously. Therefore, the Ward-Takahashi identity does not imply the relation.

In a gap region, it was proven that the Hall conductance is obtained not only from the retarded product of the current correlation function (Kubo formula), but also from the time-ordered product of it. From the time-ordered product of the current correlation function, the Hall conductance is given by the slope of $\pi^{\mu\nu}(q)$ at the origin and is written as

$$\sigma_{xy} = \frac{e^2}{3!} \epsilon^{\mu\nu\rho} \partial_\rho \pi_{\mu\nu}(q)|_{q=0}. \quad (15)$$

If the derivative ∂_ρ acts on the vertex with the external line attached, its contribution becomes zero owing to the epsilon tensor. Therefore, the case that the derivative acts on the bare propagator is survived.

By the Ward-Takahashi identity, σ_{xy} is written as a topologically invariant expression of the full propagator:

$$\begin{aligned} \sigma_{xy} &= \frac{e^2}{h} \frac{1}{24\pi^2} \int_{\text{BZ} \times S^1} d^3 p \epsilon_{\mu\nu\rho} \\ &\quad \text{tr} \left(\partial_\mu \tilde{S}^{-1}(p) \tilde{S}(p) \partial_\nu \tilde{S}^{-1}(p) \tilde{S}(p) \partial_\rho \tilde{S}^{-1}(p) \tilde{S}(p) \right) \end{aligned} \quad (16)$$

Here, the trace is taken over the Landau level index and the p_0 integral is a contour integral on a closed path.

Thus, we denote S^1 as the integration range. The integral $\frac{1}{24\pi^2} \int \text{tr}(\tilde{S}^{-1} \tilde{S})^3$ gives a integer value under general assumptions and in fact counts the number of Landau bands below the Fermi energy. Thus, the Hall conductance is proved to be a integer times e^2/h . The impurities and interactions do not modify the value of the σ_{xy} if the Fermi energy is located in the gap region or in the localized state region.

II. FLUX STATE MEAN FIELD THEORY

A. flux state on von Neumann lattice

We propose a new mean field theory based on the flux state on von Neumann lattices in this section.

The dynamical flux which is generated by interactions plays an important role in our mean field theory. Dynamics is described by a lattice Hamiltonian, which is due to the external magnetic field, and by the induced magnetic flux due to interaction, although the original electrons are defined on the continuum space. Consequently, our mean field Hamiltonian is close to the Hofstadter Hamiltonian, which is a tight-binding model with uniform constant flux. For this reason there are similarities between their solutions.

Due to the two scales of periodicity, the Hofstadter Hamiltonian exhibits interesting structure as is seen in Figure. [5] The largest gap exists along a line $\Phi = \nu\Phi_0$ with a unit of flux Φ_0 . The ground state energy becomes minimum also with this flux. These facts may suggest that the Hofstadter problem has some connection with the fractional Hall effect. We pursue a mean field theory of the condensed flux states in the quantum Hall system and point out that the Hofstadter problem is actually connected with the fractional Hall effect.

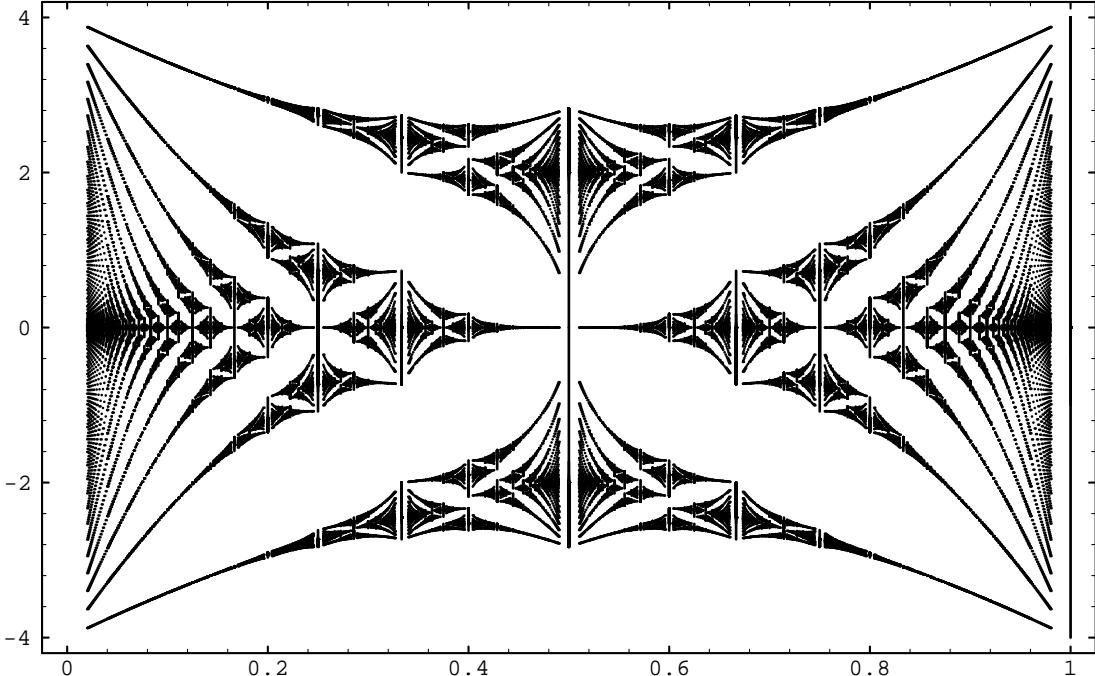


FIG. 1. The Hofstadter butterfly. Horizontal axis represents magnetic flux per plaquette. Vertical axis represents energy.

From the induced magnetic field, new Landau levels are formed. If integer number of these Landau levels are filled completely, the integer quantum Hall effect occurs. The ground state has a large energy gap and is stable against perturbations, just as in the case of the ordinary integer quantum Hall effect. We identify them as fractional quantum Hall states.

We postulate, in the quantum Hall system of the filling factor ν , the induced flux per plaquette and magnetic field of the following magnitudes:

$$\begin{aligned} \Phi_{\text{ind}} &= \nu\Phi_0, \quad \Phi_0 = \Phi_{\text{external flux}}, \\ B_{\text{ind}} &= \nu B_0, \quad B_0 = B_{\text{external magnetic field}}, \end{aligned} \tag{17}$$

where ν is the filling factor measured with the external magnetic field. We obtain a self-consistent solution with this flux. Then the integer quantum Hall effect due to the induced magnetic field could occur just at the filling factor ν , because the density satisfies the integer Hall effect condition,

$$\begin{aligned} \frac{eB_{\text{ind}}}{2\pi} N &= \frac{eB_0}{2\pi} \nu, \\ N &= 1. \end{aligned} \tag{18}$$

The ground state has a large energy gap, generally.

At the half-filling $\nu = 1/2$, the half-flux $\Phi_0/2$ is induced. We first study the state of $\nu = 1/2$, and next the states of $\nu = p/(2p \pm 1)$. At $\Phi = \Phi_0/2$, the band structure is that of a massless Dirac field and has doubling symmetry.

When an even number of Landau levels of the effective magnetic field, $B_{\text{ind}} - B_0/2$, is filled, ground states have large energy gaps. This occurs if the condition of the density,

$$\begin{aligned} \frac{e}{2\pi}|\nu - \frac{1}{2}|B_0 \cdot 2p &= \frac{eB_0}{2\pi}\nu, \\ \nu &= \frac{p}{2p \pm 1}; p, \text{ integer}, \end{aligned} \quad (19)$$

is satisfied. The factor of 2 on the left-hand side is due to the doubling of states and will be discussed later. We study these states in detail based on the von Neumann lattice representation.

The action and density operator show that there is an effective magnetic field in the momentum space. The total flux in the momentum space is in fact a unit flux. In the thermodynamic limit, in which the density in space is finite, the density in momentum space is infinite. Consequently, it is possible to make this phase factor disappear using a singular gauge transformation in the momentum space with infinitesimally small coupling.

We make a singular gauge transformation of the field in the momentum space and we have the commutation relation and the charge density.

By a Chern-Simons gauge theory in momentum space, the gauge transformation is realized. Here, the coupling constant \tilde{e} is infinitesimally small, and hence fluctuations of the Chern-Simons gauge field have a small effect, and we ignore them.

The most important part in the action in the coordinate representation is obtained and the corresponding Hamiltonian in the lowest Landau level space is given by

$$\begin{aligned} H &= -\frac{1}{2} \sum v(\mathbf{R}_2 - \mathbf{R}_1) c_0^\dagger(\mathbf{R}_1) c_0(\mathbf{R}_2) c_0^\dagger(\mathbf{R}_2) c_0(\mathbf{R}_1), \\ v(\mathbf{R}) &= \frac{\pi}{a} e^{-\frac{\pi}{2}\mathbf{R}^2} I_0\left(\frac{\pi}{2}\mathbf{R}^2\right), \end{aligned} \quad (20)$$

where I_0 is zero-th order modified Bessel function. We study a mean field solution of this Hamiltonian.

(i) Half-filled case, $\nu = 1/2$

At half-filling $\nu = 1/2$, the system has a half flux, $\Phi = \Phi_0/2$. The system, then, is described equivalently with the two-component Dirac field by combining the field at even sites with that at odd sites. We obtained self-consistent solutions numerically. As is expected, the spectrum has two minima and two zeros corresponding to doubling of states.

Around the minima, the energy eigenvalue is approximated as,

$$E(p) = E_0 + \frac{(\mathbf{p} - \mathbf{p}_0)^2}{2m^*}, \quad \mathbf{p}_0 = (0, 0), (0, \pi/a), \quad (21)$$

. The m^* in Eq.(21) is the effective mass, and is computed numerically:

$$m^* = 0.225 \sqrt{\frac{B}{B_0}} m_e, \quad B_0 = 20 \text{ Tesla}, \quad \kappa = 13, \quad \gamma = 0.914 \frac{e^2}{\kappa}. \quad (22)$$

The $\nu = 1/2$ mean field Hamiltonian is invariant under a kind of Parity, P , and anti-commutes with a chiral transformation, α_5 . If the parity is not broken spontaneously, there is a degeneracy due to the parity doublet. The doubling of the states also appears at $\nu \neq 1/2$ and plays an important role when we discuss the states away from $\nu = 1/2$ in the next part. When an additional vector potential with the same gauge, $A_x = 0$, $A_y = Bx$, is added, the Hamiltonian satisfies the properties under the above transformations, and doubling due to the parity doublet also appears. Thus, the factor of 2 is necessary in Eq.(19) and leads the principal series at $\nu = p/(2p \pm 1)$ to have the maximum energy gap.

(ii) $\nu = \frac{p}{2p \pm 1}$

If the filling factor, ν , is slightly away from 1/2, the total system can be regarded as a system with a small magnetic field of magnitude $(\nu - 1/2)B_0$. The band structure may be slightly modified. It is worthwhile to start from the band of $\nu = 1/2$ as a first approximation and to make iteration in order to obtain self-consistent solutions at arbitrary $\nu = p/(2p \pm 1)$. We solve the mean field Hamiltonian

$$\begin{aligned} H_M &= \sum U_0^{(\frac{1}{2}+\delta)}(\mathbf{R}_1 - \mathbf{R}_2) e^{i \int (\mathbf{A}^{(\frac{1}{2})} + \delta \mathbf{A}) \cdot d\mathbf{x}} v(\mathbf{R}_1 - \mathbf{R}_2) \\ &\quad c^\dagger(\mathbf{R}_1) c(\mathbf{R}_2), \\ \langle c^\dagger(\mathbf{R}_1) c(\mathbf{R}_2) \rangle_{1/2+\delta} &= U_0^{(\frac{1}{2}+\delta)} e^{i \int (\mathbf{A}^{(\frac{1}{2})} + \delta \mathbf{A}) \cdot d\mathbf{x}} \end{aligned} \quad (23)$$

under the self-consistency condition at $\nu = 1/2 + \delta$. Here we solve, instead, a Hamiltonian which has the phase of Eq.(23) but has the magnitude of the $\nu = 1/2$ state.

The integer quantum Hall state has an energy gap of the Landau levels due to $\delta\mathbf{A}$. This occurs when an integer number of Landau levels is filled completely. The Landau level structure is determined by the phase factor. Magnitudes of the physical quantities may be modified, nevertheless.

$U_0^{\frac{1}{2}}(\mathbf{R}_1 - \mathbf{R}_2)$ was obtained in the previous part, and it is approximated with the effective mass formula.

From Eq.(19), p Landau levels are completely filled, and the integer quantum Hall effect occurs at $\nu = p/(2p \pm 1)$. The energy gap is given by the Landau level spacing

$$\Delta E_{\text{gap}} = \frac{e\delta B_{\text{eff}}}{m^*} = \frac{eB_0}{m^*} \left| \nu - \frac{1}{2} \right|. \quad (24)$$

These equations were solved numerically, and the energy gaps and the widths of excited bands are obtained. Some bands are narrow and some bands are wide. Near $\nu = 1/2$, the effective magnetic field approaches zero, and the Landau level wave functions have large spatial extension. The lattice structure becomes negligible, and the spectrum shows simple Landau levels of the continuum equation in these regions. Near $\nu = 1/3$, the lattice structure is not negligible, and bands have finite widths. There are non-negligible corrections from those of continuum calculations.

Due to the energy gap of the integer Hall effect caused by the induced dynamical magnetic field, the states at $\nu = p/(2p \pm 1)$ are stable, and fluctuations are weak. Invariance under P , moreover, ensures these states to have uniform density. In systems with impurities, localized states with isolated discrete energies are generated by impurities and have energies in the gap regions. These states contribute to the density but do not contribute to the conductance. If the Fermi energy is in one of these gap regions, the Hall conductance σ_{xy} is given by a topological formula, Eq.(16), and remains constant, at $\frac{e^2}{h} \cdot \frac{p}{2p \pm 1}$. The fractional Hall effect is realized.

At a value of ν smaller than $1/3$, the Hofstadter butterfly exhibits other kinds of structures. They may be connected with the Wigner crystall.

B. Comparison with experiment

In the previous section we presented our mean field theory based on flux condensation, where lattice structure generated by the external magnetic field and condensed flux due to interaction are important ingredients. Consequently, our mean field Hamiltonian becomes very similar to that of Hofstadter, which is known to show a large energy gap zone along the $\Phi = \nu\Phi_0$ line. The line $\Phi = \nu\Phi_0$ is special in the Hofstadter problem and hence in our mean field Hamiltonian. This explains why the experiments of the fractional quantum Hall effects show characteristic behavior at $\nu = p/(2p \pm 1)$. The ground states at $\nu = p/(2p \pm 1)$ have the lowest energy and the largest energy gap, and hence these states are stable. In this section we compare the energy gaps of the principal series in the lowest order, with experiments and with Laughlin variational wave function.

The effective mass m^* of Eq.(22) was obtained from the curvature of the energy dispersion and should show a characteristic mass scale of the fractional Hall effect. The Landau level energy in the lowest approximation is calculated, and the gap energy is given in Eq.(24).

These values are compared with the experimental values and with the composite fermion mean field values. Our mean field values are close to the experimental values. For example, at $\nu = 1/3$, the composite fermion mean field theory gives $E_{\text{gap}} = 380$ [K], which is a factor of 40 larger than the experimental value. Whereas, our effective mass formula gives $E_{\text{gap}} = 24$ [K], and the other approximation gives $E_{\text{gap}} = 36$ [K], which are a factor of two or three larger than the experimental value and close to the value of the Laughlin wave function, $E_{\text{gap}} = 26$ [K]. The agreement is not perfect, but should be regarded as good as the lowest mean field approximation. Near $\nu = 1/3$, the bands have finite widths, and near $\nu = 1/2$, the widths are infinitesimal. The dependence of the width upon the filling factor, ν , and the entire structure of the bands are characteristic features of the present mean field and should be tested experimentally.

III. SUMMARY

We formulated the quantum Hall effect, integer Hall effect and fractional Hall effect with the von Neumann lattice representation of two-dimensional electrons in a strong magnetic field. The von Neumann lattice is a subset of the coherent state. The overlap of states is expressed with a elliptic theta function. They allow for a systematic method of expressing the quantum Hall dynamics.

A topological invariant expression of the Hall conductance was obtained in which compactness of the momentum space is ensured by the lattice of the coordinate space. Because the lattice has an origin in the external magnetic field, the topological character of the Hall conductance is ensured by the external magnetic field. The conductance is quantized exactly as $(e^2/h) \cdot N$ in the quantum Hall regime. The integer N increases monotonically with the chemical potential.

A new mean field theory of the fractional Hall effect that has dynamical flux condensation was proposed. In our mean field theory, lattice structure is introduced from the von Neumann lattice and, flux is introduced dynamically. The mean field Hamiltonian becomes a kind of tight-binding model, and the rich structure of the tight-binding model is seen as characteristic features of the fractional Hall effect in our mean field flux states of having a liquid property with an energy gap. These states satisfy the self-consistency condition of having the lowest energy and the largest energy gap. The physical quantities of our mean field theory are close to the experimental values in the lowest order at $\nu = p/(2p \pm 1)$.

ACKNOWLEDGEMENTS

This work was partially supported by the special Grant-in-Aid for Promotion of Education and Science in Hokkaido University provided by the Ministry of Education, Science, Sports and Culture, the Grant-in-Aid for Scientific Research on Priority Area(Physics of CP violation) (Grant No.10140201), and the Grant-in-Aid for International Scientific Research (Joint Research Grant No.10044043) from the Ministry of Education, Science, Sports and Culture, Japan.

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